Lecture 3

Number Systems

TOPICS

- Decimal Number Systems
- Binary number Systems
- Octal
- Hexadecimal
- Number conversion

- The word decimal is a derivative of decem, which is the Latin word for ten.
- Numbers today are almost universally written in a form of notation known as positional number representation.
- A number is represented as a string of digits.

- In a decimal number there are ten such digits that may be used, ranging in value from zero to nine
- Decimal numbers are positional numbers that have a *base* or *radix* of ten.
- Why ten ?

- Exactly ten distinct numerals in order to represent all possible values for each position in the number, and hence to enable us to represent all possible integer numbers in decimal notation.
- 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9 values
- For example, a number like "0123456789" has ten positions and each position can contain the digits 0-9.

- Each digit position has a *weight* associated with it.
- Each digit corresponds to a power of 10 based on its position in the number
- Number's value = a *weighted sum* of the digits

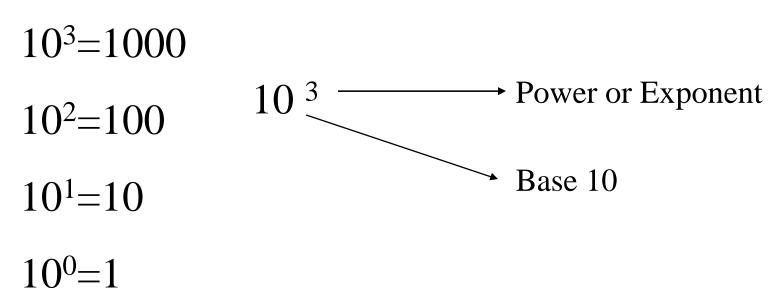
- Number's value = a *weighted sum* of the digits
- Number' value = digit * 10^x + digit * 10^x where x = (position number - 1).

It sounds complicated ?

• The powers of 10 increment from 0, 1, 2, etc. as you move right to left

| Hundreds | Tens | Units |
|----------|------|-------|
| | | |

- The BASE is 10 or Dec or simply D
- BASE = 10



• Example

$$1234_{10} = 1x \ 10^{3} + 2x \ 10^{2} + 3x10^{1} + 4x10^{0}$$
$$= 1000 + 200 + 30 + 4$$
$$= 1234_{10}$$

• Example

$$9876_{\rm D} = 9x \ 10^3 + 8x \ 10^2 + 7x10^1 + 6x10^0$$
$$= 9000 + 800 + 70 + 6$$
$$= 9876_{\rm D}$$

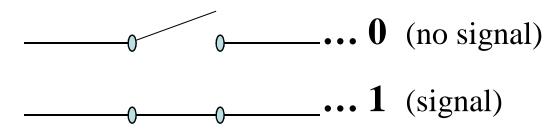
- Summary
- In the decimal system, there are 10 digits (0 through 9) which combine to form numbers as follows:
 - 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 ...

- The term computer numbering formats refers to the schemes implemented in <u>digital computer</u> and <u>calculator</u> hardware and software to represent <u>numbers</u>
- Digital Computers and Calculator use a binary systems
 - Why binary?

- Easy to represent binary values electrically
- Voltages and currents.
- Can be implemented using circuits
- Create the building blocks of modern computers

- Reliability
 - With only 2 values, can be widely separated, therefore clearly differentiated
 - Binary numbers are made of binary digits

• Binary DigiTs (BITs) can be represented electronically:



Base is 2 or 'b' or 'B' or 'Bin'

Two symbols: 0 and 1

Each place is weighted by the power of 2

- All the information in the digital computer is represented as bit patterns
- What is a bit pattern?

• This is one bit 01010101 This is called as the bit pattern

- Look at this bit pattern 0101 0101
- How many bits are present ?
- Count the number of ones and the zeros in the above pattern

Answer = Total 8 bits

0101 0101

This pattern is represented as follows in the digital computer

| Bit7 | Bit 6 | Bit 5 | Bit 4 | Bit 3 | Bit 2 | Bit 1 | Bit 0 |
|------|-------|-------|-------|-------|-------|-------|-------|
| 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 |

- A single bit can represent two <u>states</u>:0 1
- Therefore, if you take two bits, you can use them to represent four unique states:

00, 01, 10, & 11

• And, if you have three bits, then you can use them to represent eight unique states: 000, 001, 010, 011, 100, 101, 110, & 111

•With every bit you add, you double the number of states you can represent. Therefore, the expression for the number of states with n bits is 2^n . Most computers operate on information in groups of 8 bits,

| Bit7 | Bit 6 | Bit 5 | Bit 4 | Bit 3 | Bit 2 | Bit 1 | Bit 0 |
|------|-------|-------|-------|-------|-------|-------|-------|
| 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 |

- There are 8 bits in the above table
- Group of 4 bits = 1 Nibble
- Group of 8 bits = 1 Byte
- Group of 16 bits = 1 Word 2 Bytes = 1 Word

| Bit7 | Bit 6 | Bit 5 | Bit 4 | Bit 3 | Bit 2 | Bit 1 | Bit 0 |
|------|-------|-------|-------|-------|-------|-------|-------|
| 0 | 1 | 0 | 1 | 0 | 1 | 0 | 1 |

- There are 8 bits in the above table
- Bit 0 is called the Least Significant Bit LSB
- Bit 1 is called the Most Significant Bit MSB

Bit positions and their values

| Bit7 | Bit 6 | Bit 5 | Bit 4 | Bit 3 | Bit 2 | Bit 1 | Bit 0 |
|------|-------|-------|-------|-------|-------|-------|-------|
| 27 | 26 | 25 | 24 | 23 | 22 | 21 | 20 |
| 128 | 64 | 32 | 16 | 8 | 4 | 2 | 1 |

Convert Decimal to Binary

- 1. Divide decimal number by the base (e.g. 2), remainder is 0 or 1.
- 2. The *remainder* is the Lowest Significant Bit (LSB).
- 3. Repeat first two steps until no *divisor* remains. means divide quotient by 2, marking down the remainder each time. Last remainder is Most Significant Bit (MSB).
- 4. Write the remainder in reverse order

Convert Decimal to Binary

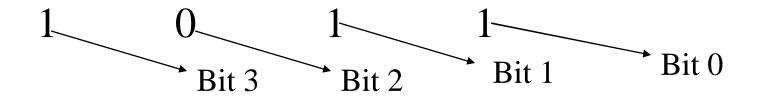
Example: Convert Decimal 13 (13 $_{10}$) to Binary : Repeated division by 2 (till quotient is zero)

| Divide-by -2 | Quotient | Remainder | Binary Bits |
|--------------|----------|-----------|-------------|
| 13/2 | 6 | 1 | Bit 0 = 1 |
| 672 | 3 | 0 | Bit 1 = 0 |
| 372 | 1 | 1 | Bit 3 = 1 |
| 1+72 | 0 | 1 | Bit 4 = 1 |

Answer = 1101_2

Convert Binary to Decimal Example: Convert 1101₂

Multiply each 1 bit by the appropriate power of 2 and add them together.



Convert Binary to Decimal

Example: Convert 1101₂

| Bit7 | Bit 6 | Bit 5 | Bit 4 | Bit 3 | Bit 2 | Bit 1 | Bit 0 | |
|---|-------|----------------|-------|-------|-------|-------|-------|--|
| 27 | 26 | 2 ⁵ | 24 | 23 | 22 | 21 | 20 | |
| 128 | 64 | 32 | 16 | 8 | 4 | 2 | _ 1 | |
| | | | | 1 | T | 0 | 1 | |
| | | | | | | | | |
| Multiply with $8 \times 1 + 4 \times 1 + 2 \times 0 + 1 \times 1 = 8 + 4 + 0 + 1$ | | | | | | | | |

these values

= 13

Example: $1011_2 \text{ or } 1011_B$ $= 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$ = 8 + 0 + 2 + 1 $= 11_{10}$ Binary Number Systems Example: $10110_2 \text{ or } 10110_B$ $1 \ge 2^4 + 0 \ge 2^3 + 1 \ge 2^2 + 1 \ge 2^1 + 0 \ge 2^0$ = 16 + 0 + 4 + 2 + 0 $= 22_{10}$

Other Number Systems

- Octal and hex are a convenient way to represent binary numbers, as used by computers.
- Computer mechanics often need to write out binary quantities, but in practice writing out a binary number such as

Other Number Systems

- 100100110100001 is tedious, and prone to errors.
- Therefore, binary quantities are written in a base-8 ("octal") or, much more commonly, a base-16 ("hexadecimal" or "hex") number format.

Octal Number Systems

- Base = 8 or 'o' or 'Oct'
- 8 symbols: { 0, 1, 2, 3, 4, 5, 6, 7 }
- Example 123, 567, 7654 etc 987 This is incorrect why?
- How to represent a Decimal Number using a Octal Number System ?

Octal Number Systems

- Repeated Division by 8
- Example

$$213_{10} = ()_8$$
?

| Divide-by -8 | Quotient | Remainder | Octal digit |
|--------------|----------|-----------|-------------------|
| 213 / 8 | 26 | 5 | Lower digit $= 5$ |
| 26 / 8 | 3 | 2 | Second digit =2 |
| 3 / 8 | 0 | 3 | Third digit =3 |

Answer = 325_8

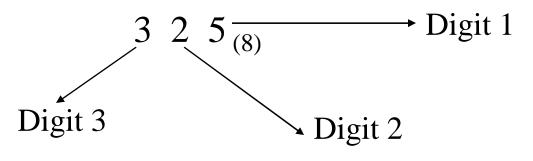
Octal Number Systems

- How to convert 325₈ back to Decimal ?
- Use this table and multiply the digits with the position values

| Digit 8 | Digit 7 | Digit 6 | Digit 5 | Digit 4 | Digit 3 | Digit 2 | Digit 1 |
|------------|------------|------------|------------|-----------------------|------------|------------|------------|
| 87 | 86 | 85 | 84 | 8 ³ | 82 | 81 | 80 |
| | ••••• | 32768 | 4096 | 512 | 64 | 8 | 1 |

Octal Number Systems

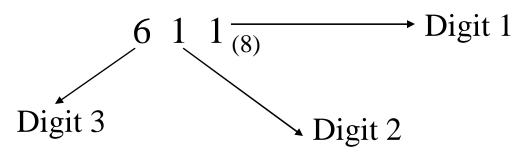
- How to convert 325₈ back to Decimal ?
- Consider the above number



$$3 \times 8^{2} + 2 \times 8^{1} + 5 \times 8^{0} = 3 \times 64 + 2 \times 8 + 5 \times 1$$
$$= 192 + 16 + 5$$
$$= 213$$

Octal Number Systems

- Example Convert 611₈
- Consider the above number



 $6 \times 8^{2} + 1 \times 8^{1} + 1 \times 8^{0} = 6 \times 64 + 1 \times 8 + 1 \times 1$ = 384 + 8 + 1= 393

Octal Number Systems

• Convert 393 to octal

| Divide-by -8 | Quotient | Remainder | Octal digit |
|--------------|----------|-----------|-------------------|
| 393 / 8 | 49 | 1 | Lower digit $= 1$ |
| 49 / 8 | 6 | 1 | Second digit =1 |
| 6 / 8 | 0 | 6 | Third digit =6 |

Answer = 611_8

<u>Hexadecimal Number</u> <u>Systems</u>

• Base = 16 or 'H' or 'Hex'

16 symbols: { 0, 1, 2, 3, 4, 5, 6, 7,8,9 }

{ 10=A, 11=B, 12=C, 13=D, 14=E, 15=F}

<u>Hexadecimal Number</u> <u>Systems</u>

- {0,1,2,3,4,5,6,7,8,9,A,B,C,D,E,F} It uses 6 Letters !
- Example AB12, 876F, FFFF etc
- How to represent a Decimal Number using a Hexadecimal Number System ?

Hex Number Systems

- Repeated Division by 16
- Example

$$213_{10} = ()_{16}$$
?

| Divide-by -16 | Quotient | Remainder | Hex digit |
|---------------|----------|-----------|-------------------|
| 213 / 16 | 13 | 5 | Lower digit $= 5$ |
| 13 / 16 | 0 | 13 | Second digit =D |
| | | | |

Answer = $D5_{16}$

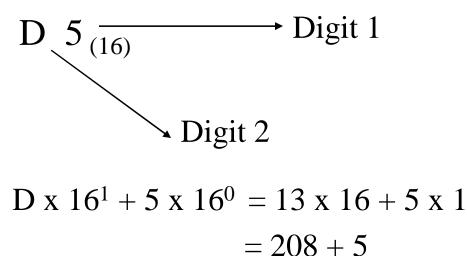
Hex Number Systems

- How to convert $D5_{16}$ back to Decimal ?
- Use this table and multiply the digits with the position values

| Digit 8 | Digit 7 | Digit 6 | Digit 5 | Digit 4 | Digit 3 | Digit 2 | Digit 1 |
|------------|-----------------|-----------------|------------|-----------------|-----------------|-----------------|-----------------|
| 167 | 16 ⁶ | 16 ⁵ | 164 | 16 ³ | 16 ² | 16 ¹ | 16 ⁰ |
| | | | ••••• | 4096 | 256 | 16 | 1 |

Hex Number Systems

- How to convert $D5_{16}$ back to Decimal ?
- Consider the above number

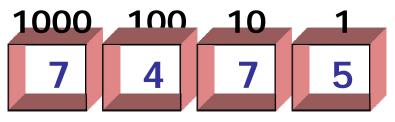


Number System

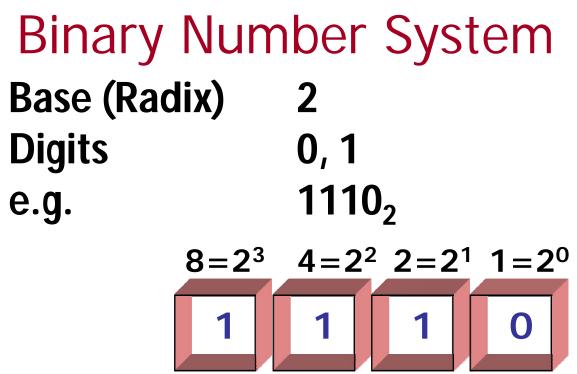
- Each number system has a number of different digits which is called the radix or the base of the number system.
 - Decimal Base = 10
 - Binary Base = 2
 - Octal Base = 8
 - Hexadecimal (Hex) Base = 16

Decimal Number System Base (Radix) 10 Digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9 e.g. 7475₁₀

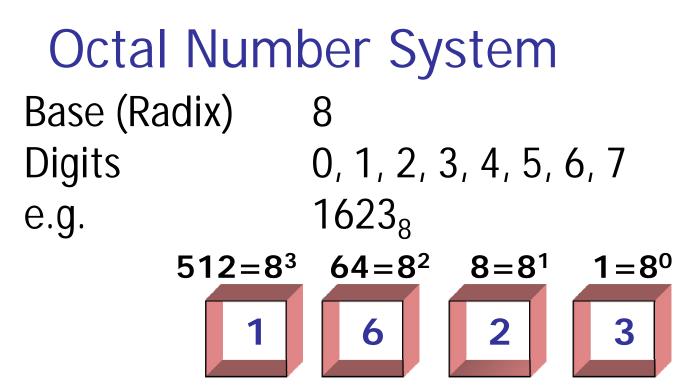
The magnitude represented by a digit is decided by the position of the digit within the number.



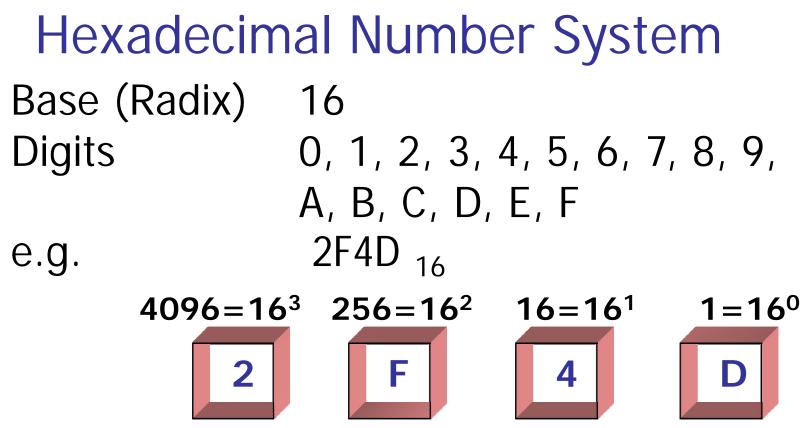
For example the digit 7 in the left-most position of 7475 counts for 7000 and the digit 7 in the second position from the right counts for 70.



The digit 1 in the third position from the right represents the value 4 and the digit 1 in the fourth position from the right represents the value 8.



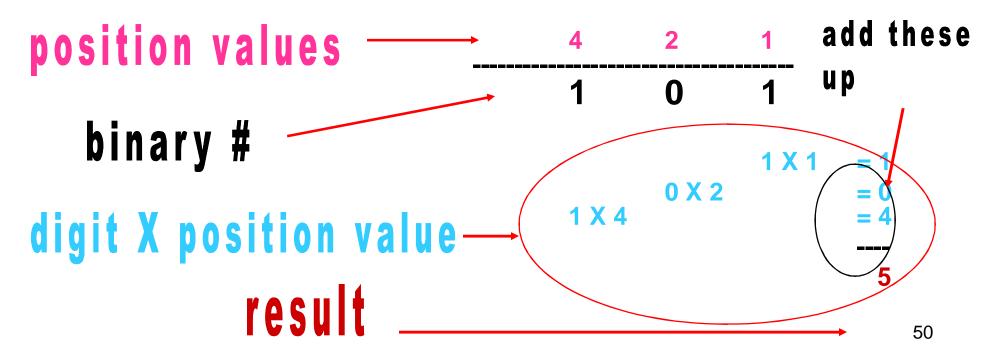
The digit 2 in the second position from the right represents the value 16 and the digit 1 in the fourth position from the right represents the value 512.



The digit F in the third position from the right represents the value 3840 and the digit D in the first position from the right represents the value 1.

Converting from binary to decimal

- Each position for a binary number has a value.
- For each digit, multiply the digit by its position value
- Add up all of the products to get the final result
- The decimal value of binary 101 is computed below:

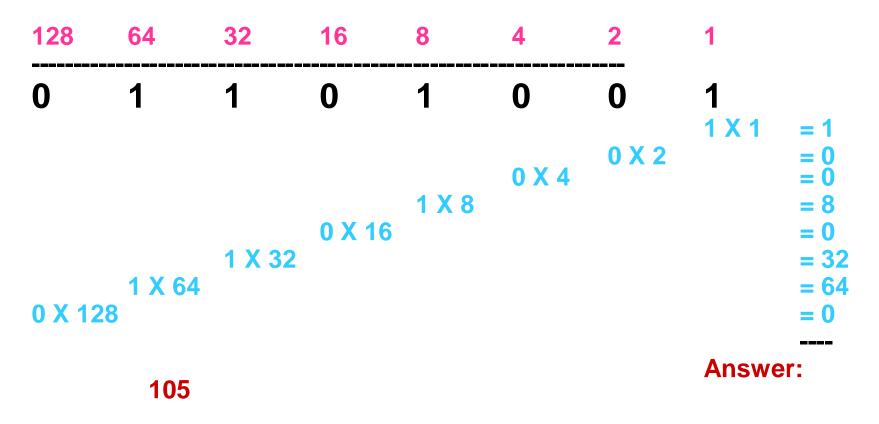


What about a longer number?

- In general, the "position values" in a binary number are the powers of two.
 - The first position value is 2^0 , i.e. one
 - The 2nd position value is 2¹, i.e. two
 - The 2nd position value is 2^2 , i.e. four
 - The 2nd position value is 2³, i.e. eight
 - The 2nd position value is 2^4 , i.e. sixteen
 - etc.
- Example on next slide

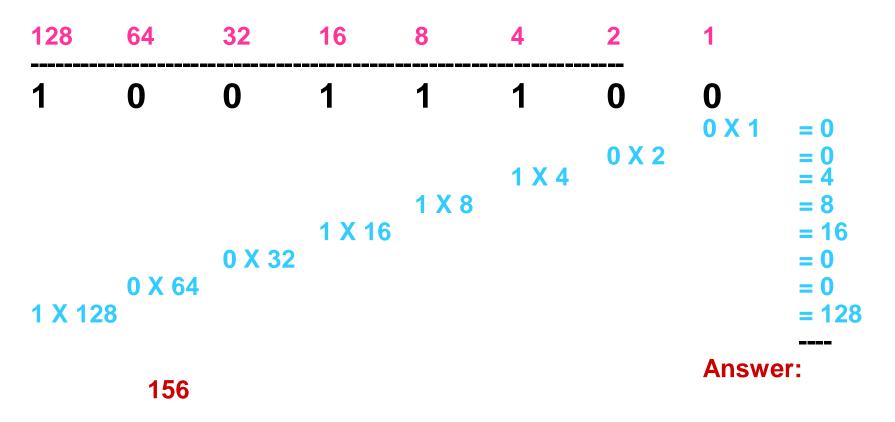
Example

• The value of binary 01100001 is decimal 105. This is worked out below:



Another example

• The value of binary 10011100 is decimal 156. This is worked out below:



Some Terminology

- The following are some terms that are used in the computer field
 - Each digit of a binary number is called a bit.
 - A binary number with eight bits (i.e. digits) is called a byte.

How many different numbers?

- There are two different binary numbers with one bit:
 0
 1
- There are four different binary numbers with two bits:
 - 00 (i.e. decimal 0)
 - 01 (i.e. decimal 1)
 - 10 (i.e. decimal 2)
 - 11 (i.e. decimal 3)

How many different numbers?

- There are eight different binary numbers with three bits:
 - 000 (i.e. decimal 0)
 - 001 (i.e. decimal 1)
 - 010 (i.e. decimal 2)
 - 011 (i.e. decimal 3)
 - 100 (i.e. decimal 4)
 - 101
 - 110

- 111

(i.e. decimal 6)

(i.e. decimal 5)

(i.e. decimal 7)

different numbers - General

• For n bits there are $2^n R$ of the entire of the second secon

| <u># of bits</u> | # of different binary numbers |
|------------------|-------------------------------|
| 1 bit: | $2^1 = 2$ |
| 2 bits: | $2^2 = 4$ |
| 3 bits: | $2^3 = 8$ |
| 4 bits: | $2^4 = 16$ |
| 5 bits: | $2^5 = 32$ |
| 6 bits: | $2^6 = 64$ |
| 7 bits: | $2^7 = 128$ |
| 8 bits: | $2^8 = 256$ |
| 9 bits: | $2^9 = 512$ |
| 10 bits: | $2^{10} = 1024$ |
| etc. | |

Smallest value for a binary

- The smallest value for a binary number of any number of bits is zero.
- This is the case when all bits are zero:

Smallest value for a binary

• The smallest value for a binary number with any number of bits is zero (i.e. when all the bits are zeros)

| # of bits | smallest binary # | decimal value |
|-----------------|-------------------|---------------|
| 1 bit: | 0 | 0 |
| 2 bits: | 00 | 0 |
| 3 bits: | 000 | 0 |
| 4 bits: | 0000 | 0 |
| 5 bits: | 00000 | 0 |
| 6 bits: | 000000 | 0 |
| 7 bits: | 0000000 | 0 |
| 8 bits: etc. | 0000000 | 0 |

Largest value for a binary

- The largest value for a binary number with a specific number of bits (i.e. digits) is when all of the bits are one.
- General rule: for a binary number with n bits, the largest possible value is : 2ⁿ - 1

Largest numbers

• The following are the largest values for binary numbers with a specific number of bits:

| # of bits | largest binary # | decimal value |
|-----------------|------------------|---------------|
| 1 bit: | 1 | 1 |
| 2 bits: | 11 | 3 |
| 3 bits: | 111 | 7 |
| 4 bits: | 1111 | 15 |
| 5 bits: | 11111 | 31 |
| 6 bits: | 111111 | 63 |
| 7 bits: | 111111 | 127 |
| 8 bits: etc. | 11111111 | 255 |

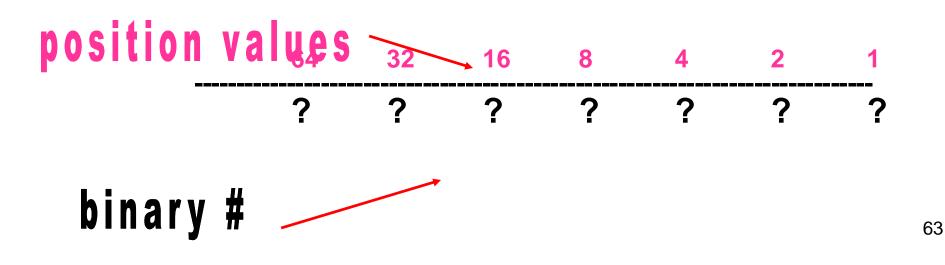
Converting a decimal# to a binary# (1)

- Step 1: figure out how many bits you will need (see the chart on the previous slide).
 - Example 1: To convert the decimal number 16 to binary, you will need at least 5 bits.
 (With 4 bits you can only store numbers up to 15 but with 5 bits you can store numbers up to 31)

 Example 2: To convert the decimal number 106 to binary, you will need at least 7 bits.
 (With 6 bits you can only store numbers up to 63 but with 7 bits you can store numbers up to 127)

Converting a decimal# to a binary# (2)

- Step 2: Keep a chart of the position values and the digits for your binary number. At first you will not know what any of the digits will be.
- Example: convert decimal 106 to binary
 - step 1 : figure out that you need 7 bits (see earlier slides)
 - step 2 : keep track of position values and bits for binary #



Converting a decimal # to binary (3)

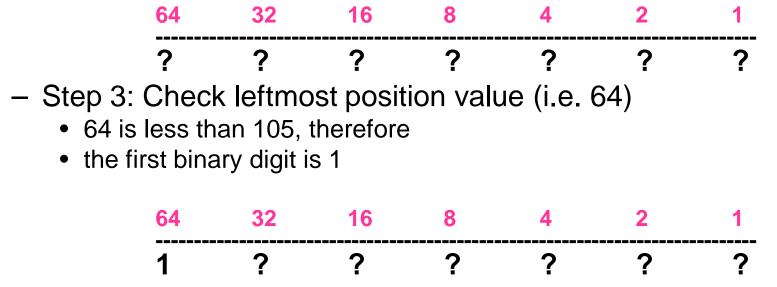
- Step 3: starting with the leftmost digit, see if the position value is greater than, less than or equal to the number you are trying to convert.
 - if the position value is greater than the number then
 - make the binary digit in that position zero
 - if the position value is less than the number then
 - make the binary digit in that position one
 - subtract the position value from the decimal # you are trying to convert
 - if the position value is equal to the number then
 - make the binary digit in that position one
 - make the rest of the binary digits zero
 - you are done

Converting a decimal # to binary (4)

Step 4: do step 3 again with the next digit.
 Keep doing this until you've figured out all of the digits.

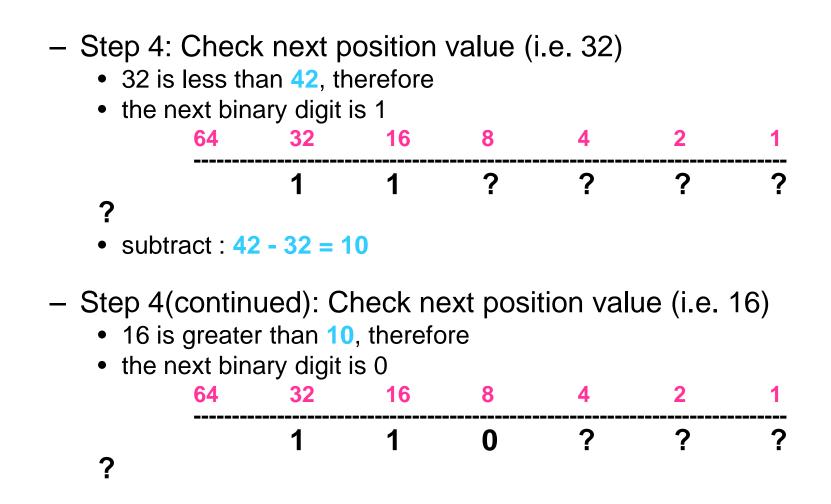
Example

- Example: convert decimal 106 to binary
 - Step 1: You need 7 bits (see earlier slides for explination).
 - Step 2: keep track of position values for bits

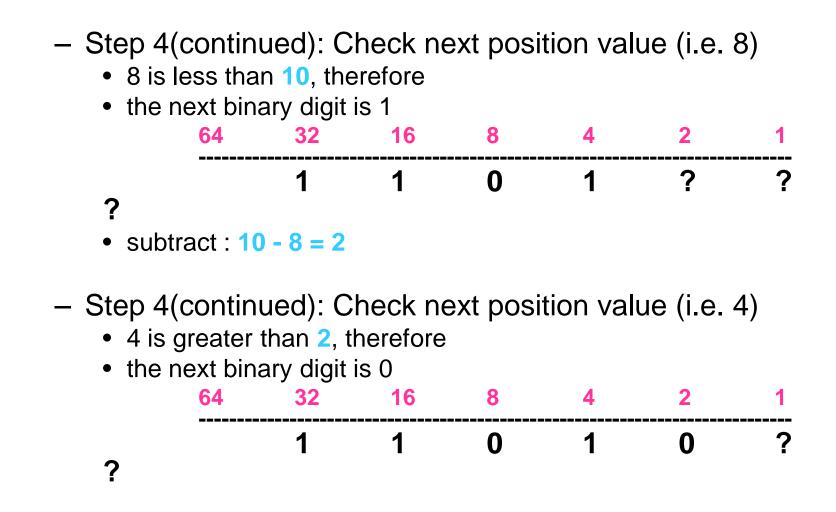


• subtract : 106 - 64 = 42

Example (continued-2)

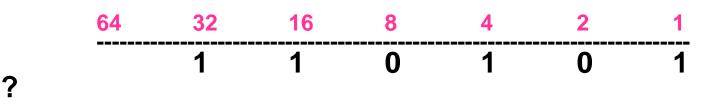


Example (continued-3)

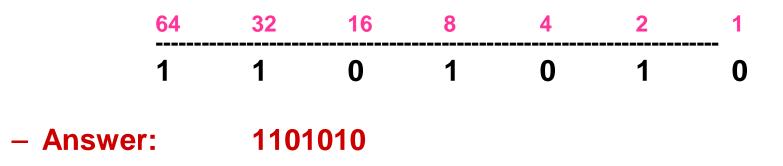


Example (continued-4)

- Step 4(continued): Check next position value (i.e. 2)
 - 2 is equal to 2, therefore
 - the next binary digit is 1



• Since the position value was equal to the number (i.e. 2) the rest of the binary digits are all zeros



Dec → Binary : Another Method

Example:

Convert the decimal number 26₁₀ into its binary equivalent.

Solution:

$$2 \overline{\smash{\big)}} \frac{13}{26} \quad r = 0 \leftarrow LSB$$

$$2 \overline{\smash{\big)}} \frac{6}{13} \quad r = 1$$

$$2 \overline{\smash{\big)}} \frac{3}{6} \quad r = 0$$

$$2 \overline{\smash{\big)}} \frac{1}{3} \quad r = 1$$

$$2 \overline{\smash{\big)}} \frac{1}{3} \quad r = 1$$

$$r = 1 \leftarrow MSB$$

Contd...

Example:

Convert the decimal number 41_{10} into its binary equivalent.

Solution:

$$2 \int \frac{20}{41} \quad r = 1 \leftarrow LSB$$

$$2 \int \frac{10}{20} \quad r = 0$$

$$2 \int \frac{5}{10} \quad r = 0$$

$$2 \int \frac{2}{5} \quad r = 1$$

$$2 \int \frac{1}{2} \quad r = 0$$

$$2 \int \frac{1}{2} \quad r = 1 \leftarrow MSB$$

Dec → Binary : More Examples

- a) $13_{10} = ?$ 1101₂
- b) $22_{10} = ?$ 10110₂
- c) $43_{10} = ?$ 101011₂
- d) $158_{10} = ?$ 10011110₂

Binary → Decimal Conversion

Example:

Convert the binary number 10010₂ into its decimal equivalent.

Solution:

| 1 | | 0 | | 0 | | 1 | | 0 | | |
|-----------------------|---|-----------------------|---|-----------------------|---|-----------------------|---|-----------------------|---|-------------------------|
| 2 ⁴ | | 2 ³ | | 2 ² | | 2 ¹ | | 2 ⁰ | | |
| 16 | | 8 | | 4 | | 2 | | 1 | | |
| 16 | + | 0 | + | 0 | + | 2 | + | 0 | = | 18 ₁₀ |

Binary \rightarrow Dec : Example 2

Example:

Convert the binary number 0110101₂ into its decimal equivalent.

Solution:

| 0 | | 1 | | 1 | | 0 | | 1 | | 0 | | 1 | | |
|-----------------------|---|-----------------------|---|-----------------------|---|-----------------------|---|-----------------------|---|-----------------------|---|----------------|---|-------------------------|
| 2 ⁶ | | 2 ⁵ | | 2 ⁴ | | 2 ³ | | 2 ² | | 2 ¹ | | 2 ⁰ | | |
| 64 | | 32 | | 16 | | 8 | | 4 | | 2 | | 1 | | |
| 0 | + | 32 | + | 16 | + | 0 | + | 4 | + | 0 | + | 1 | = | 53 ₁₀ |

∴0110101₂ = 53₁₀

Binary → Dec : More Examples

- a) 0110 ₂ = ?
- b) $11010_2 = ?$
- c) 0110101 $_2 = ?$
- d) 11010011₂ = ?

Summary & Review



- a) Divide the Decimal Number by 2; the remainder is the LSB of Binary Number.
- b) If the Quotient Zero, the conversion is complete; else repeat step (a) using the Quotient as the Decimal Number. The new remainder is the next most significant bit of the *Binary Number*.

BINARY Weighted Multiplication Base Decimal

- a) Multiply each bit of the *Binary Number* by it corresponding bit-weighting factor (i.e. Bit-0→2⁰=1; Bit-1→2¹=2; Bit-2→2²=4; etc).
- b) Sum up all the products in step (a) to get the *Decimal Number*.

Why is it called "binary" (or base-2)?

- The prefix "bi" means "two" in Latin
- Binary derives its name from the fact that the digits in a "Binary" number can only have two possible values, 0 or 1
- It is also called "base-2" based on the fact that the column values are the powers of 2.
 (i.e. 2⁰ 2¹ 2² 2³ 2⁴ 2⁵ etc.)

Hexadecimal (AKA "Hex") numbers

What is a hexadecimal or base-16 number.

- A "hexadecimal" number is a number where each digit may be one of sixteen possible values.
- The possible values for a hexadecimal digit are: 0 1 2 3 4 5 6 7 8 9 A B C D E F
- A digit of "A" stands for the number 10 "B" stands for the number 11 "C" stands for the number 12 "D" stands for the number 13 "E" stands for the number 14 "F" stands for the number 15
- Keep reading ...

Hexadecimal numbers

- The following are all valid hexadecimal nubmers
 - A
 - -9(yes, a hexadecimal number does not HAVE TO contain letters)
 - 1001 (yes, a hexadecimal number does not HAVE TO contain letters)
 - 9C5
 - BFE
 - Etc.
- To understand what a specific hexadecimal number means, you can convert it into an equivalent decimal number. (see next slide)

Converting a Hexadecimal number to Decimal

• The value of hexadecimal A12F is decimal 41,263. See below:

| 4096 (i.e 16 ³) | 256 (i.e 16 ²) | 16 (i.e 16 ¹) | 1 (i.e 16 ⁰) |
|-----------------------------|----------------------------|---------------------------|--------------------------|
| Α | 1 | 2 | F |
| | | | 15 X 1 = 15 |
| | | 2 X 16 | = 32 |
| | 1 X 256 | | = 256 |
| 10 X 4096 | | | = 40,960 |
| | | | |
| | | | Answer: |
| 41,263 | | | |

Hex numbers are a "shorthand" for binary nubmers

- It is very easy to convert between Hex and binary numbers.
- Each Hex number is 1/4th the length of its equivalent binary number.
- Therefore Hex is often used as a "shorthand" for writing an equivalent binary number.
- Keep reading ...

Assignment-1

- Convert the decimal number 2610.75 into its binary, octal and hexadecimal equivalent ?
- Convert 1011011.1001 into decimal, octal and hexadecimal number system.